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Behaviour of distributional generalized Fourier Kontorovich-Lebedev Transform

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ABSTRACT

This paper deals with the distributional generalized Fourier-Kontorovich-Lebedev transform, FKF transform of distributional function f in dual space of $FKF_{a,b,\alpha}^{\Omega}$ i.e. $FKF\,'^{\Omega}_{a,b,\alpha}$. We have discussed some properties of FKF transform. In this paper we have also extended the properties for analytic behaviour on FKF transform and discussed the results for different dimensions.

Keywords: Macdonald function, Distributional generalized Fourier-Kontorovich-Lebedev transform, Analytic behaviour.

INTRODUCTION

It is quite well known that there are several problems which can be solved by repeated applications of the Fourier transform and Kontorovich- Lebedev transform. If we construct an integral transform for which kernel is the product of kernels of the Fourier transform and Kontorovich-Lebedev transform, namely Fourier-Kontorovich-Lebedev transform, then without successive use we easily deal with problems occurring in two variables.

The conventional two-sided Fourier-Kontorovich-Lebedev transformation is defined by

$$F(s, q) = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(t, x), \frac{e^{-ist} K_{iq}(x)}{\sqrt{x}}\, dt\, dx$$

for a suitably restricted function $f(t, x)$ on $\Omega = \{(t, x)/-\infty < t < \infty,\ 0 < x < \infty\}$ and $K_{iq}(x)$ is Macdonald function (Lebedev 1965) where q and x are positive variables.

The notation and terminology of the work will follow that of Zemanian [3].

2. Testing function space $FKF_{a,b,\alpha}^{\Omega}$ and its dual :

All infinitely differentiable complex valued functions $\varphi(t, x)$ defined over

$$\Omega = \{(t, x)/-\infty < t < \infty,\ 0 < x < \infty\}$$

is said to be a member of the space $FKF_{a,b,\alpha}^{\Omega}$ if it is smooth and for each non negative integer $k = (k_1, k_2) \in R^2$

$$\gamma_{a,b,\alpha,k}(\varphi) = \sup_{\substack{-\infty<t<\infty t \\ 0<x<\infty}} \left|\eta(t,x) D_t^{k_1} \Delta_x^{k_2} \varphi(t,\, x)\right|$$
$$\leq C_q A^k k^{ka},\ k = 0,\, 1,\, 2$$

where $D_t^{k_1} = \dfrac{\partial^{k_1}}{\partial x^{k_1}}$ and $\Delta_x^{k_2} = \left(y^2 D_y^2 + 2yD_y - y^2\right)^{k_2}$ and the constants C_q and A depend on the testing function $\varphi(t,\, x)$.

For $k = 0$, we get $k^{ka} = 1$. The topology of the space $FKF_{a,\,b,\,\alpha}^{\Omega}$ is the space generated by the countable multinorm given by

$$S = \bigcup_{\substack{k_1=-\infty \\ k_2=0}}^{\infty} \left\{\gamma_{a,b,\alpha,k}\right\}.$$

With this topology $FKF_{a,\,b,\,\alpha}^{\Omega}$ is a countably multinorm space. A sequence $\{\varphi_\nu\}$ is said to converge in $FKF_{a,\,b,\,\alpha}^{\Omega}$ to φ if for each non negative integer $l,\ q$,

$\gamma_{a,b,\alpha,k}(\varphi_\nu - \varphi) \to 0$ as $\nu \to \infty$.

3. Some Properties on the space $FKF_{a,\,b,\,\alpha}^{\Omega}$:

One can easily see that $FKF_{a,\,b,\,\alpha}^{\Omega}$ is a linear space and $\gamma_{a,b,\alpha,k}$ for each non-negative integer $k \in R^2$ is a seminorm while $\gamma_{a,b,\alpha,0}$ is a norm. Therefore the collection of seminorms $\left\{\gamma_{a,b,\alpha,k}\right\}_{k=0}^{\infty}$ is separating. We equip $FKF_{a,\,b,\,\alpha}^{\Omega}$ with the topology generated by seminorm $\left\{\gamma_{a,b,\alpha,k}\right\}_{k=0}^{\infty}$ thereby making $FKF_{a,\,b,\,\alpha}^{\Omega}$ a countably multinormed space. A sequence $\{\varphi_m(t,\, x)\}_{m=1}^{\infty}$ in $FKF_{a,\,b,\,\alpha}^{\Omega}$ converges to $\varphi(t,\, x)$ iff for each k, $\gamma_{a,b,\alpha,k}(\varphi_m - \varphi_n) \to 0$ as m and n tends to ∞ independent of each other.

It can be readily seen that $FKF_{a,\,b,\,\alpha}^{\Omega}$ is a sequentially complete locally convex Hausdorff space.

We shall list few properties of $FKF_{a,\,b,\,\alpha}^{\Omega}$

i) $D(\Omega) \subset FKF_{a,\,b,\,\alpha}^{\Omega}$ and converges in $D(\Omega)$ implies the convergence in $FKF_{a,\,b,\,\alpha}^{\Omega}$.

Consequently, the restriction of any $f \in FKF\ '^{\Omega}_{a,\,b,\,\alpha}$ to $D(\Omega)$ is in $D'(\Omega)$. Hence the member of $FKF_{a,\,b,\,\alpha}^{\Omega}$ are called distributions in the sence of Zemanian [3].

ii) If $f(t,\, x)$ is locally integrable function defined on Ω such that

$$\int_{-\infty}^{\infty}\int_{0}^{\infty} \left[\eta_{a,\,b}(t,x)\right]^{-1} f(t,x)\ dt\ dx$$

exists then $f(t,x)$ generates a regular generalized function in $FKF\ '^{\Omega}_{a,\,b,\,\alpha}$ defined as

$$\langle f,\, \varphi\rangle = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(t,x)\ \varphi(t,x)\ dt\ dx, \quad \text{where } \varphi \in FKF_{a,\,b,\,\alpha}^{\Omega}$$

iii) For every fixed $s,\ \alpha > 0,\ q > 0$ where $a \le \operatorname{Re} s \le b$ and $\frac{e^{ist} K_{iq}(x)}{\sqrt{x}}$ is a member of $FKF^{\Omega}_{a,\,b,\,\alpha}$

4. The distributional two sided Fourier-Kontorovich-Lebedev transform :

For $f(t,\ x) \in FKF\ '^{\Omega}_{a,\,b,\,\alpha}$ there exists a unique open region $\wedge f$ in upper $(t,\ x)$ plane and $\Omega_f = \{(P,\ Q)/\sigma_1 < \operatorname{Re} s < \sigma_2,\ 0 < q < \infty\}$ where σ_1 and σ_2 are infimum and supremum of $\wedge f$. Then the distributional two sided Fourier-Kontorovich-Lebedev transform $F(s,\ q)$ of $f(t,\ x)$ is defined as a conventional function on Ω_f by

$$F(s,\ q) = [(FKF)f]_{(s,\ q)} = \left\langle f(t,\ x),\ \frac{e^{-ist} K_{iq}(x)}{\sqrt{x}} \right\rangle,\ (s,\ q) \in \Omega_f$$

where Ω_f is called as region of definition for $(FKF)f(t,\ x)$. For any fixed $(s,\ q) \in \Omega_f$ the right hand side of above equation has a meaning as the application of $f(t,x)$ to $\frac{e^{-ist} K_{iq}(x)}{\sqrt{x}} \in FKF^{\Omega}_{a,\,b,\,\alpha}$ for fixed $\alpha > 0,\ q > 0$ and for any α and q such that $\sigma_1 < a \le \operatorname{Re} s \le b < \sigma_2$.

Analyticity Theorem :

If $(FKF)f = F(s,\ q)$ for $(s,\ q) \in \Omega_f$ then $F(s,\ q)$ is analytic on Ω_f and

$$D^k F(s,\ q) = \left\langle f(t,\ x),\ D^k \frac{e^{-ist} K_{iq}(x)}{\sqrt{x}} \right\rangle,\ (s,\ q) \in \Omega_f, \text{where } D^k = \frac{\partial^{|k|}}{\partial^{k_1} \partial^{k_2}},\quad |k| = k_1 + k_2 .$$

Proof : Let $D^k F(s,\ q) - \left\langle f(t,\ x),\ D^k \frac{e^{-ist} K_{iq}(x)}{\sqrt{x}} \right\rangle = \left\langle f(t,\ x),\ \psi_{\Delta s \Delta y}(t,\ x) \right\rangle$

Then $\psi_{\Delta s \Delta y}(t,\ x) \in FKF^{\Omega}_{a,\,b,\,\alpha}$ as $|\Delta s| \to 0$ and $|\Delta x| \to 0$.

Hence $\psi_{\Delta s \Delta y}(t,\ x)$ converges in $FKF^{\Omega}_{a,\,b,\,\alpha}$ to zero. Consequently $\left|\eta(t,x) D_t^{k_1} \Delta_x^{k_2} \psi_{\Delta s \Delta y}(t,\ x)\right|$ converges in $FKF^{\Omega}_{a,\,b,\,\alpha}$ to zero as $|\Delta s| \to 0$ and $|\Delta x| \to 0$, which proves theorem.

CONCLUSION

Fourier-Kontorovich-Lebedev transform is one of the flourishing field of active research due to its wide range of applications. F-K-L transformation is applied to a larger spaces of generalized functions and extended the theory to solve Dirichlet problem for a wedge with generalized boundary conditions.

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