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Effect of viscous dissipation on unsteady periodic MHD poiseuille flow with transpiration cooling and thermal radiation

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ABSTRACT

This paper deals with the effects of heat and mass transfer on unsteady periodic flow of a viscous incompressible and electrically conducting fluid in a horizontal channel taking into consideration the effect of internal heating by viscous dissipation. The lower stationary plate and the upper plate in unsteady periodic motion are subjected to a same constant injection and suction velocity respectively. The temperature of the upper plate in periodic motion various periodically with time. The flow in the channel is also acted upon by periodic variation of pressure gradient. A magnetic field of uniform strength is applied in the direction normal to the plates. A closed form solution of the problem is obtained. The effects of various flow parameters on the velocity, temperature and concentration fields have been discussed. The skin friction and rate of heat and mass transfer are obtained. The results are presented and discussed with the help of graphs.

Keywords: MHD, transpiration cooling, thermal radiation, viscous dissipation, porous channel.

Nomenclature

C^*	fluid concentration
Cp	specific heat at constant pressure
D	molecular diffusivity
Ec	Eckert number
k	thermal conductivity
M	Hartman number
N	radiation parameter
Nu	Nusselt number
p	constant pressure
p^*	pressure
Pr	Prandtl number
q_r	radiative heat flux
Re	Reynolds number
Sh	Sherwood number
Sc	Schmidt number
T^*	fluid temperature
t^*	time
$u^*(y^*,t^*)$	axial velocity
y	non-dimensional width of the channel
Greek symbols	
α	mean radiation absorption coefficient
ρ	fluid density
σ	electric conductivity
υ	kinematic viscosity
ω^*	frequency of oscillations
τ	skin friction

INTRODUCTION

The most sensitive fluid property to temperature rise is the fluid viscosity. Fluid viscosity due to temperature may affect the flow characteristics as well as the efficient operation of industrial machinery where lubrication is important. Viscous dissipation heat cannot be neglected for fluids with high Prandtl number or flow at high gravitational field. In [8] the effect of viscous dissipation in natural convection is appreciable when the induced kinetic energy becomes appreciable in comparison to the amount of heat transferred. When a fluid is sheared some of the work done is dissipated as heat and the shear-induced heating definitely results to temperature increase within the fluid. Bister and Emanuel [5] recognised the viscous dissipation of turbulent kinetic energy as a significant heat source in hurricanes and showed that this heat source increases the efficiency of the hurricane. Barletta [2] concluded that in the case of upward flow, dimensionless velocity and temperature are increasing functions of the viscous dissipation parameter, while in the case of downward flow, velocity decreases despite the temperature increase with viscous dissipation. Zanchini [13] concluded that in case of asymmetric heating, viscous dissipation enhances the effects of flow reversal in downward flow, while it lowers this effect in case of upward flow. Barletta and Rossi di Schio [3] investigated the effect of viscous dissipation in a vertical tube with uniform heat flux and concluded that in a buoyancy assisted flow, the dimensionless velocity close to the wall of the duct increases as Brinkman number increases. In a related work Barletta and Zanchini [4] were used two different perturbation expansions to study the fully developed laminar mixed convection in an inclined channel and it was reported that the presence of viscous dissipation leads to a sharp dependence of the dimensionless pressure drop coefficient and the Nusselt number on the Grashof number. The effects of heat generation and absorption on natural convection flow between two infinite vertical parallel plates that are subjected to periodic heating in the presence of suction and injection were studied by Jha and Ajibade [9], where it was shown that the influence of heat generation and that of absorption on flow are reciprocal.

In the present paper, we study the effect of viscous dissipation on unsteady periodic MHD poiseuille flow with transpiration cooling and thermal radiation in the presence of heat and mass transfer.

Mathematical Formulation

Consider an unsteady MHD flow of an electrically conducting, viscous incompressible fluid in a horizontal channel distance 'd' apart. A Cartesian co-ordinate system is introduced such that x^*-axis lies along the centerline of the channel and y^*-axis is perpendicular to the wall of the channel. A magnetic field of uniform strength B_0 is taken to be acting along the y^*-axis. The fluid is injected through the lower stationary porous plate and sucked through the upper porous plate in oscillating motion in its own plane. The injection and suction velocity at both plates is constant and is given by V. The Reynolds number is assumed to be very small so that the induced magnetic field is negligible. All the physical quantities are independent of x^* for this problem of fully developed laminar flow. The governing equations for the present physical situation following [12] and taking into account viscous dissipation heating within the fluid are

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^*}{\rho} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho Cp}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho Cp}\frac{\partial q^*}{\partial y^*} + \frac{\mu}{\rho Cp}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{4}$$

It is assumed that the fluid is optically thin with relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2 T^* \tag{5}$$

The boundary conditions relevant to the problem are

$$y^* = d/2 : u^* = U\cos\omega^* t^*, v^* = V, T^* = T_0 \cos\omega^* t^*, C^* = C_0 \cos\omega^* t^*, \tag{6}$$

$$y^* = -d/2 : u^* = 0, v^* = V, T^* = 0, C^* = 0. \tag{7}$$

For the oscillatory internal flow in the channel the periodic pressure gradient variations are assumed to be of the form

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = P\cos\omega^* t^* \tag{8}$$

because the problem of assumption of constant injection and suction velocity V at the upper and lower plates respectively, continuity equation (1) integrates to

$$v^* = V \tag{9}$$

substituting Eq. (9) and introducing the following non-dimensional parameters

$$x = \frac{x^*}{d}, y = \frac{y^*}{d}, u = \frac{u^*}{U}, T = \frac{T^*}{T_0}, t = \omega^* t^*, \omega = \frac{\omega^* d^2}{\upsilon}, p = \frac{p^*}{\rho U V}, C = \frac{C^*}{C_0}, Sc = \frac{\upsilon}{D} \tag{10}$$

into Eqs. (2), (3) and (4), we get

$$\frac{\omega}{\mathrm{Re}}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{\mathrm{Re}}u \tag{11}$$

$$\frac{\omega}{\mathrm{Re}}\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} = \frac{1}{\mathrm{RePr}}\frac{\partial^2 T}{\partial y^2} - \frac{N^2}{\mathrm{RePr}}T + \frac{Ec}{\mathrm{Re}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

$$\frac{\omega}{\mathrm{Re}}\frac{\partial C}{\partial t} + \frac{\partial C}{\partial y} = \frac{1}{\mathrm{ReSc}}\frac{\partial^2 C}{\partial y^2} \tag{13}$$

where $\mathrm{Re} = \frac{Vd}{\upsilon}, \mathrm{Pr} = \frac{\mu Cp}{k}, N = 2\alpha\frac{d}{\sqrt{k}}, Ec = \frac{U^2}{CpT_0}$

The boundary conditions in dimensionless form become

$$u = \cos t, T = \cos t, C = \cos t \quad at \quad y = \frac{1}{2} \tag{14}$$

$$u = 0, T = 0, C = 0 \quad at \quad y = -\frac{1}{2} \tag{15}$$

1. Solution of the problem

The mathematical solution of this periodic flow in the porous channel when the fluid is also acted upon by a periodic drop in pressure, we assume the solution in the complex notations as

$$u(\mathrm{y,t}) = \mathrm{u}_0(\mathrm{y})\mathrm{e}^{it}, \mathrm{T}(\mathrm{y,t}) = \theta_0(\mathrm{y})\mathrm{e}^{it}, \mathrm{C}(\mathrm{y,t}) = \phi_0(\mathrm{y})\mathrm{e}^{it}, -\frac{\partial p}{\partial x} = pe^{it}. \tag{16}$$

The real part of the solution will have physical significance.

The boundary conditions (14) and (15) can also be written in complex notations as

$$u = e^{it}, \mathrm{T} = e^{it}, \mathrm{C} = e^{it}, \text{ at } \mathrm{y} = \frac{1}{2} \tag{17}$$

$$u = 0, T = 0, C = 0 \;\; at \;\; y = -\frac{1}{2} \tag{18}$$

Substituting Eq. (16) into Eqs. (11), (12) and (13), we get

$$u_0'' - \mathrm{Re}\, u_0' - (\mathrm{i}\,\omega + \mathrm{M}^2) u_0 = -P\,\mathrm{Re} \tag{19}$$

$$\theta_0'' - \mathrm{Re}\,\mathrm{Pr}\,\theta_0' - (\mathrm{i}\,\omega\mathrm{Pr} + \mathrm{N}^2)\theta_0 = -Ec\,\mathrm{Pr}\, u_0'^2 \tag{20}$$

$$\phi_0'' - \mathrm{Re}\, Sc\phi_0' - (\mathrm{i}\,\omega\mathrm{Sc})\phi_0 = 0 \tag{21}$$

These ordinary differential equations denote differentiation with respect to y

The boundary conditions (17) and (18) reduce to

$$u_0 = 1, \theta_0 = 1, \phi_0 = 1 \;\; at \;\; y = \frac{1}{2} \tag{22}$$

$$u_0 = 0, \theta_0 = 0, \phi_0 = 0 \;\; at \;\; y = -\frac{1}{2} \tag{23}$$

The solutions of Eqs. (19), (20) and (21) under the boundary conditions (22) and (23) are obtained as

$$u(\mathrm{y,t}) = \left[c + \frac{1}{\sinh\left(\frac{m-n}{2}\right)} \left\{ c\left(e^{my} \sinh\left(\frac{n}{2}\right) - e^{ny} \sinh\left(\frac{m}{2}\right)\right) + \frac{1}{2}\left(e^{my-\frac{n}{2}} - e^{ny-\frac{m}{2}} \right) \right\} \right] e^{it} \tag{24}$$

$$T(\mathrm{y,t}) = \mathrm{A}_3\, e^{ry} + A_4 e^{sy} + h_1 e^{2my} + h_2 e^{2ny} + h_3 e^{(\mathrm{m+n})y} \tag{25}$$

$$C(\mathrm{y,t}) = \left(\frac{e^{ay+\frac{a}{2}} - e^{by+\frac{b}{2}}}{e^a - e^b} \right) e^{it} \tag{26}$$

where the constants $c,\ m,\ n,\ r,\ s,\ A_1,\ A_2,\ A_3,\ A_4,\ h_1,\ h_2,\ h_3,\ a,\ b$ are defined in the appendix. If the effect of viscous dissipation is neglected ($Ec = 0$), the results presented in Eqs. (24)-(26) coincide with the result of [12].

From the velocity field obtained in Eq. (24) we can find skin-friction τ at the lower plate as

$$\tau = \left(\frac{\partial u_0}{\partial y} \right)_{y=-\frac{1}{2}} e^{it} = |F| \cos(\mathrm{t} + \varphi) \tag{27}$$

with $|F| = \sqrt{F_r^2 + F_i^2}$ and $\varphi = \tan^{-1} \dfrac{F_i}{F_r}$

where $F_r + iF_i = \dfrac{1}{\sinh\left(\frac{m-n}{2}\right)} \left\{ c\left(me^{-\frac{m}{2}} \sinh\frac{n}{2} - ne^{-\frac{n}{2}} \sinh\frac{m}{2} \right) + \frac{m-n}{2} e^{-\frac{m+n}{2}} \right\}$

From the temperature field obtained in Eq. (25) we can find Nusselt number Nu at the lower plate as

$$Nu = \left(\frac{\partial \theta_0}{\partial y} \right)_{y=-\frac{1}{2}} e^{it} = |H| \cos(\mathrm{t} + \psi) \tag{28}$$

with $|H| = \sqrt{H_r^2 + H_i^2}$ and $\psi = \tan^{-1} \dfrac{H_i}{H_r}$

where $H_r + iH_i = \dfrac{r-s}{e^r - e^s} e^{it}$

From the concentration field obtained in Eq. (26) we can find Sherwood number Sh at the lower plate as

$$Sh = \left(\frac{\partial C_0}{\partial y}\right)_{y=-\frac{1}{2}} e^{it} = |G|\cos(t+\theta) \tag{29}$$

with $|G| = \sqrt{G_r^2 + G_i^2}$ and $\theta = \tan^{-1}\dfrac{G_i}{G_r}$

where $G_r + iG_i == \dfrac{a-b}{e^a - e^b} e^{it}$

RESULTS AND DISCUSSION

An unsteady periodic MHD poiseuille flow of viscous incompressible fluid is considered in a horizontal channel with transpiration cooling and thermal radiation. The influence of viscous dissipation heat on the flow is also considered. The problem is presented in graphical form in Figs. 1 to 7 so as to clearly reveal the influence of each of the governing parameters on the flow behaviors.

From Fig. 1, it can be seen that velocity profile decreases with increase in magnetic field and frequency of oscillations. The increasing strength of the magnetic field physically means that the Lorentz force similar to drag force increase leads to increase the drag force which slows down the motion of fluid, hence velocity decreases. Also this figure indicates that velocity increases with increase in Reynolds number representing injection/suction through the lower and the upper plates of the channel respectively. Physically, it means that the increase of the injection/suction velocity strengthens of the flow in the channel.

It is observed from Fig. 2 that temperature profile decreases with increase in Reynolds number, frequency of oscillations and Prandtle number. In presence of injection/suction more amount of fluid is pushed into the flow field due to which the flow field suffers a decrease in temperature of the flow field at all points. With the increase of Prandtl number, the thermal diffusivity decreases and this phenomenon lead to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer.

The effect of viscous dissipation and radiation on the temperature of the flow field is shown in Fig. 3. The temperature of the flow field is found to decrease with the increase in viscous dissipative heat in terms of Eckert number and radiation parameter. Viscous dissipation as a source term in the energy equation converts kinetic motion of the fluid to thermal energy and give rise to a change in the temperature distribution. The effect of radiation decrease the rate of energy transport to the fluid, thereby decreasing the fluid temperature.

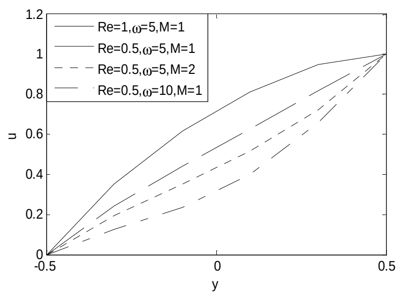

Fig.1 Variation of velocity for different Re, ω, M **with** $\Pr = 5,\ t = 0$

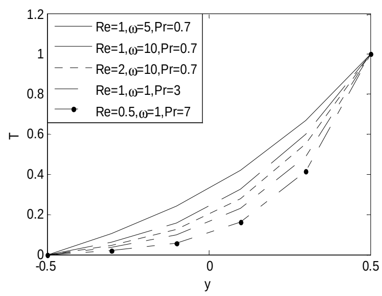

Fig.2. Variation of temperature for different $\mathrm{Re}, \omega, \Pr$ **with** $E = 0, N = 1, t = 0$

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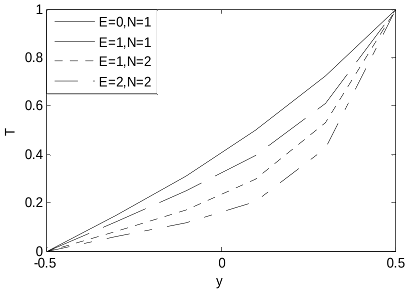

Fig.3. Variation of temperature for different E, N with $\mathrm{Re}=0.5,\ \mathrm{Pr}=0.71,\ \omega=1,\ t=0$

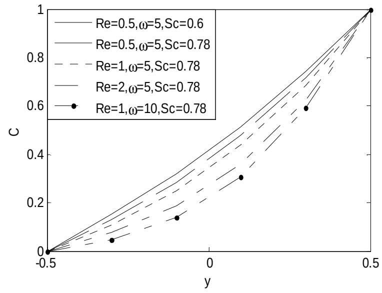

Fig.4. Variation of concentration for different Re, ω, Sc **with** $t=0$

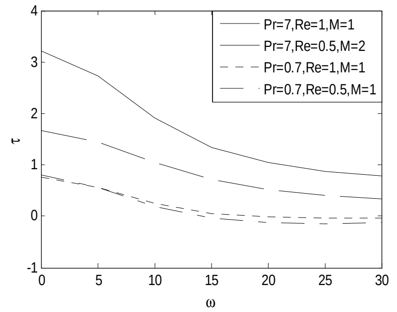

Fig.5. Shear stress versus ω for different $\mathrm{Pr},\ \mathrm{Re},\ \mathrm{M}$

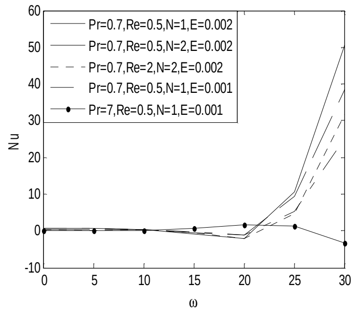

Fig.6. Rate of heat transfer versus ω for different $\mathrm{Pr},\ \mathrm{Re},\ N, E$

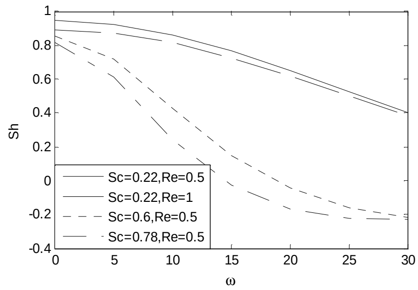

Fig.7. Rate of mass transfer versus ω for different Sc, Re

Concentration profiles are shown in Fig. 4 for different values of the various parameters involved. Figure 4 illustrate that concentration field decreases with the increase in Reynolds number, frequency of oscillation and Schmidt number. Higher values of Schmidt number amounts to a fall in the chemical molecular diffusivity i.e. less diffusion therefore takes place by species transfer causing a reduction in concentration.

The skin friction at the lower plate versus frequency of oscillations is depicted in Fig. 5. It is clear from the figure that coefficient of skin friction decreases with an increase in Hartman number whereas it increases with Prandtl number and Reynolds number.

Figure 6 illustrates the variation of rate of heat transfer versus frequency of oscillation for various parameters. It is clear from the figure that rate of heat transfer at the lower plate of the channel decreases with an increase in Prandtl number, Reynolds number and radiation parameter whereas it increases with Eckert number.

The variation of rate of mass transfer versus frequency of oscillation for various parameters is depicted in Fig. 7. It is clear from the figure that rate of mass transfer at the lower plate of the channel decreases with an increases in Sc and Re.

Appendix

$$c=\frac{\Pr\mathrm{Re}}{i\omega+M^2},\quad m=\frac{R+\sqrt{R^2+4\left(i\omega+M^2\right)}}{2},\quad n=\frac{R-\sqrt{R^2+4\left(i\omega+M^2\right)}}{2},$$

$$A_1=\frac{\left(e^{-\frac{n}{2}}-2c\sinh\frac{n}{2}\right)}{2\sinh\left(\frac{m-n}{2}\right)},\quad A_2=\left(-c-A_1e^{-\frac{m}{2}}\right)e^{\frac{n}{2}},$$

$$h_1=\frac{-Ec\Pr A_1^2m^2}{4m^2-2\mathrm{Re}\Pr m-\left(i\omega\Pr+N^2\right)},\quad h_2=\frac{-Ec\Pr A_2^2n^2}{4n^2-2\mathrm{Re}\Pr n-\left(i\omega\Pr+N^2\right)},$$

$$h_3=\frac{-Ec\Pr 2A_1A_2mn}{(m+n)^2-\mathrm{Re}\Pr(m+n)-\left(i\omega\Pr+N^2\right)},$$

$$r=\frac{1}{2}\left(\mathrm{Re}\Pr+\sqrt{\mathrm{Re}^2\Pr^2+4\left(i\omega+N^2\right)}\right),\quad s=\frac{1}{2}\left(\mathrm{Re}\Pr-\sqrt{\mathrm{Re}^2\Pr^2+4\left(i\omega+N^2\right)}\right),$$

$$A_3=\frac{e^{\frac{r}{2}}}{e^r-e^s}\left(1+h_1\left(e^{s-m}-e^m\right)+h_2\left(e^{s-n}-e^n\right)+h_3\left(e^{s-\frac{m+n}{2}}-e^{\frac{m+n}{2}}\right)\right),$$

$$A_4=-e^{\frac{s}{2}}\left(A_3e^{-\frac{r}{2}}+h_1e^{-m}+h_2e^{-n}+h_3e^{-\frac{m+n}{2}}\right),$$

$$a=\frac{1}{2}\left(\mathrm{Re}\,Sc+\sqrt{\mathrm{Re}^2Sc^2+4\left(i\omega Sc\right)}\right),\quad b=\frac{1}{2}\left(\mathrm{Re}\,Sc-\sqrt{\mathrm{Re}^2Sc^2+4\left(i\omega Sc\right)}\right).$$

CONCLUSION

This work is extension of Swapna and Varma [12] in which they were analyzed the fluid flow and heat and mass transfer without viscous dissipation effect. When the viscous dissipation term is neglected in this work, there is an excellent agreement with the results of [12]. From the present study, the concluding remarks have been taken as follows:

- The magnetic parameter retards the velocity at all points of the flow field and also skin friction.
- Velocity and skin friction increases with increase in Reynolds number whereas temperature field, concentration field and the rate of heat and mass transfer decreases.
- Frequency of oscillations reduces the velocity, temperature and concentration field.
- Prandtl number decelerates the temperature field and rate of heat transfer whereas enhances skin friction.
- Radiation parameter diminishes temperature field and rate of heat transfer.
- Eckert number reduces the temperature field whereas enhances rate of heat transfer.
- Schmidt number reduces the concentration field and rate of mass transfer.

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