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Fully developed free convective flow of a Williamson fluid in a vertical channel under the effect of a magnetic field

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INTRODUCTION

The analysis of free convection in vertical channels occurs in many industrial processes and natural phenomena. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the related papers on this topic, such as Aung and Worku (1986), Cheng et al. (1990), Barletta (1998,1999), El-Din (2003), Boulama and Galanis (2004), Barletta et al. (2005) deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. In all the above studies of free and mixed convection flow in vertical channels are based on the hypothesis that the fluids are Newtonian. However, because of their fundamental and technological importance, theoretical studies of free, forced and mixed convection flow of non-Newtonian fluids in channels and tubes are very important in several industrial processes. Szeri and and Rajagopal (1985) have studied the flow of a third grade fluid between heated parallel plates caused by external pressure gradient and obtained similarity solutions of the energy equation, numerically. Akyıldız (2001) have studied the flow of third grade fluid between heated parallel plates. Chamka et al. (2002) have studied the fully developed free connective flow of - micropular fluid between two vertical parallel plates analytically. Recently, Siddiqui et al. (2010) have investigated the flow of a third grade non-Newtonian fluid between two parallel plates separated by a finite gap by using the Adomian decomposition method. Williamson fluid is characterized as a non-Newtonian fluid with shear thinning property, i.e., viscosity decreases with increasing rate of shear stress (Dapra and Scarpi, 2007).

The use of electrically conducting fluids under the influence of magnetic fields in various industries has led to a renewed interest in investigating hydromagnetic flow and heat transfer in different geometries. For example, Sparrow and Cess (1961) considered the effect of a magnetic field on the free convection heat transfer from a surface. Garandet et al. (1992) have studied buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Chamkha (1999) have investigated free convection effects on three-dimensional flow over a vertical stretching surface in the presence of a magnetic field. Bhargava et al. (2003) have studied the effect of magnetic field on the free convection flow of micropolar fluid between two parallel porous vertical plates. Hayat et al. (2004) have studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. Hazeem attia (2005) have investigated the unsteady flow of a dusty conducting fluid between parallel porous plates. Sanyal and Adhikari (2006) have studied the effects of radiation on MHD fluid flow in vertical channel.

In view of these, we studied the fully developed free convection flow of a Williamson fluid in a vertical channel under the effect of magnetic field. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are studied through graphs in detail.

2. Mathematical formulation
The equations governing the flow of an incompressible Williamson fluid are given by

$$\nabla.V = 0 \tag{2.1}$$

$$\rho\frac{dV}{dt} = \rho f + \nabla.\tau \tag{2.2}$$

where ρ denotes the constant fluid density, V is the velocity vector and f represents the body force per unit mass. The operator d/dt denotes the material time derivative and τ is the stress tensor.

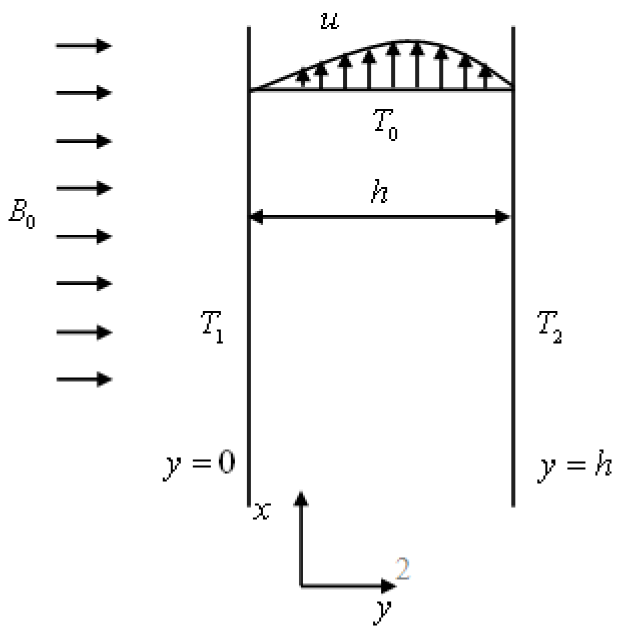

Fig. 1 The physical model

The constitutive equation for a Williamson fluid is given by

$$\tau = -\left[\eta_\infty + (\eta_0 + \eta_\infty)(1 - \Gamma\dot{\gamma})^{-1}\right]\dot{\gamma} \tag{2.3}$$

Where τ is the extra stress tensor, η_∞ is the infinite shear rate, viscosity η_o is the zero shear rate viscosity, Γ is the time constant and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{2.4}$$

where π is the second invariant stress tensor. We consider in the constitutive Eq. (2.3) the case for which $\eta_\infty = 0$ and $\Gamma\dot{\gamma} < 1$ so we can write.

$$\tau = -\eta_0(1 + \Gamma\dot{\gamma})\dot{\gamma} \tag{2.5}$$

The above model reduces to Newtonian for $\Gamma = 0$

We consider the laminar free convection flow of a Williamson fluid between two plates at distance h a apart, as shown in Fig.1.We choose co-ordinates system, with X - axis parallel to the flow while Y - axis is normal to the flow. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The flow assume steady and fully developed, i.e., the transverse velocity is zero. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in asymmetric heating situation under these assumptions the equations that describe the physical situation are

$$\mu \frac{d^2u}{dy^2} + \Gamma \frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - \sigma B_0^2 u + \rho g \beta (T - T_0) = 0 \tag{2.6}$$

$$\frac{\partial^2 T}{\partial y^2} = 0 \tag{2.7}$$

where σ is the electrical conductivity.

Subject to the boundary conditions

$$u(0) = 0, \quad T(0) = T_1, \quad u(h) = 0, \quad T(h) = T_2 \tag{2.8}$$

Introducing the following non-dimensional variables

$$\bar{u} = \frac{u}{U}, \bar{y} = \frac{y}{h}, \bar{x} = \frac{x}{h}, We = \frac{U\Gamma}{\eta_0 h}, \theta = \frac{T - T_0}{T_2 - T_0}, r_T = \frac{T_1 - T_0}{T_2 - T_0} \tag{2.9}$$

into Eqs. (2.6) and (2.7), we get (after dropping the bars)

$$\frac{d^2u}{dy^2} + We \frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - M^2 u + \frac{Gr}{\mathrm{Re}}\theta = 0 \tag{2.10}$$

$$\frac{d^2\theta}{dy^2} = 0 \tag{2.11}$$

where $M = B_0 h \sqrt{\frac{\sigma}{\eta_0}}$ is the Hartmann number, $Gr = \frac{g\beta(T_2 - T_0)h^3}{\nu^2}$ is the Grashof number and $\mathrm{Re} = \frac{Uh}{\nu}$ is the Reynolds number.

The corresponding dimensionless boundary conditions

$$u(0) = 0, \quad \theta(0) = r_T, \quad u(1) = 0, \quad \theta(1) = 1 \tag{2.12}$$

3. Perturbation Solution

Eq. (2.10) is non-linear and it is difficult to get a closed form solution. However for vanishing We, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Nevertheless, small Γ suggests the use of perturbation technique to solve the non-linear problem. Accordingly, we write

$$u = u_0 + We u_1 \tag{3.1}$$

and

$$\theta = \theta_0 + We\theta_1 \tag{3.2}$$

Substituting equations (2.11) and (2.12) into Eqs. (2.8) and (2.9) and boundary conditions (2.10) and then equating the like powers of We, we obtain

3.1 Zeroth-order system $\left(We^0\right)$

$$\frac{d^2u_0}{dy^2} - M^2 u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 \tag{3.3}$$

$$\frac{d^2\theta_0}{dy^2} = 0 \tag{3.4}$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0, \quad \theta_0(0) = r_T, \quad \theta_0(1) = 1 \tag{3.5}$$

3.2 First-order system (We)

$$\frac{d^2u_1}{dy^2} - M^2u_0 = -\frac{d}{dy}\left[\left(\frac{du_0}{dy}\right)^2\right] - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{3.6}$$

$$\frac{d^2\theta_1}{dy^2} = 0 \tag{3.7}$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0, \quad \theta_1(0) = 0, \quad \theta_1(1) = 0 \tag{3.8}$$

3.3 Zeroth-order solution

Solving Eqs. (3.3) and (3.4) using the boundary conditions (3.8), we get

$$\theta_0 = r_T + (1 - r_T)y \tag{3.9}$$

$$u_0 = \frac{Gr}{\mathrm{Re}}\frac{1}{M^2}\left[A_1 \sinh My - r_T \cosh My + (1 - r_T)y + r_T\right] \tag{3.10}$$

here $A_1 = \dfrac{r_T \cosh M - 1}{\sinh M}$.

3.4 First-order solution

Solving Eq. (3.7) subject to the boundary conditions in Eq. (3.8), we get

$$\theta_1 = 0 \tag{3.11}$$

Substituting the Eqs. (3.10) and (3.11) into the Eq. (3.6) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = \left(\frac{Gr}{\mathrm{Re}}\right)^2 \frac{1}{M^4}\left[\begin{array}{l} -\dfrac{A_3}{3M^2}\cosh My + A_7 \sinh My - \dfrac{A_2}{3M^2}\sinh 2My \\ +\dfrac{A_3}{3M^2}\cosh 2My + \dfrac{A_4}{2M}y\sinh My - \dfrac{A_5}{2M}y\cosh My \end{array}\right] \tag{3.12}$$

where $A_2 = M^3(r_T^2 + A_1^2)$, $A_3 = 2A_1 r_T M^3$, $A_4 = 2r_T(1 - r_T)M^2$, $A_5 = 2A_1(1 - r_T)M^2$,

$$A_6 = \frac{A_2 \sinh 2M}{3M^2} - \frac{A_3 \cosh 2M}{3M^2} - \frac{A_4 \sinh M}{2M} + \frac{A_5 \cosh M}{2M},$$

$$A_7 = \left[A_6 + \frac{A_3 \cosh M}{3M^2}\right] \Big/ \sinh M.$$

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \theta_0 + \Gamma\theta_1 = \theta_0 = r_T + (1 - r_T)y \tag{3.13}$$

and $$u = u_0 + \Gamma u_1 \tag{3.14}$$

RESULTS AND DISCUSSION

Fig. 2 shows the effect of Weissenberg number We on u for $M = 1$, $r_T = 0.5$, $Gr = 1$ and $\mathrm{Re} = 1$. It is observed that, velocity u first decreases and then increases with increasing We.

The effect of Hartman number M on u for $We = 0.1$, $r_T = 0.5$, $Gr = 1$ and $\mathrm{Re} = 1$ is represented in Fig. 3. It is found that, the velocity u decreases with an increase in Hartmann number M.

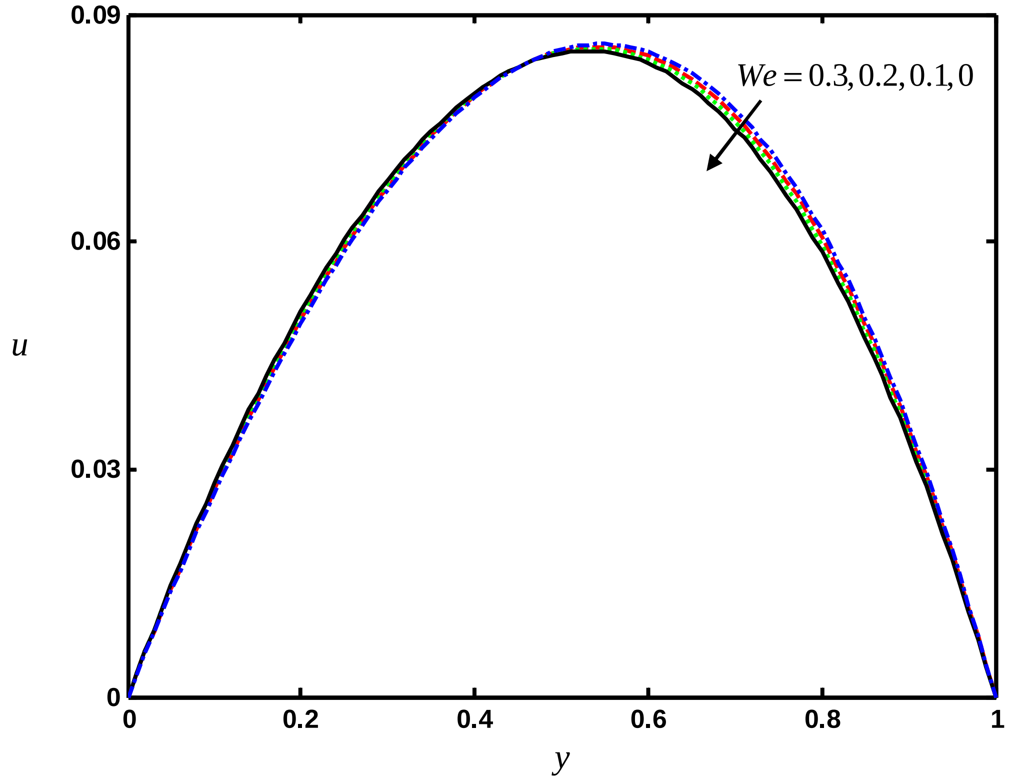

Fig. 2. Effect of Weissenberg number We **on** u **for** $Gr=1$, $M=1$, $r_T=0.5$ **and** $\mathrm{Re}=1$.

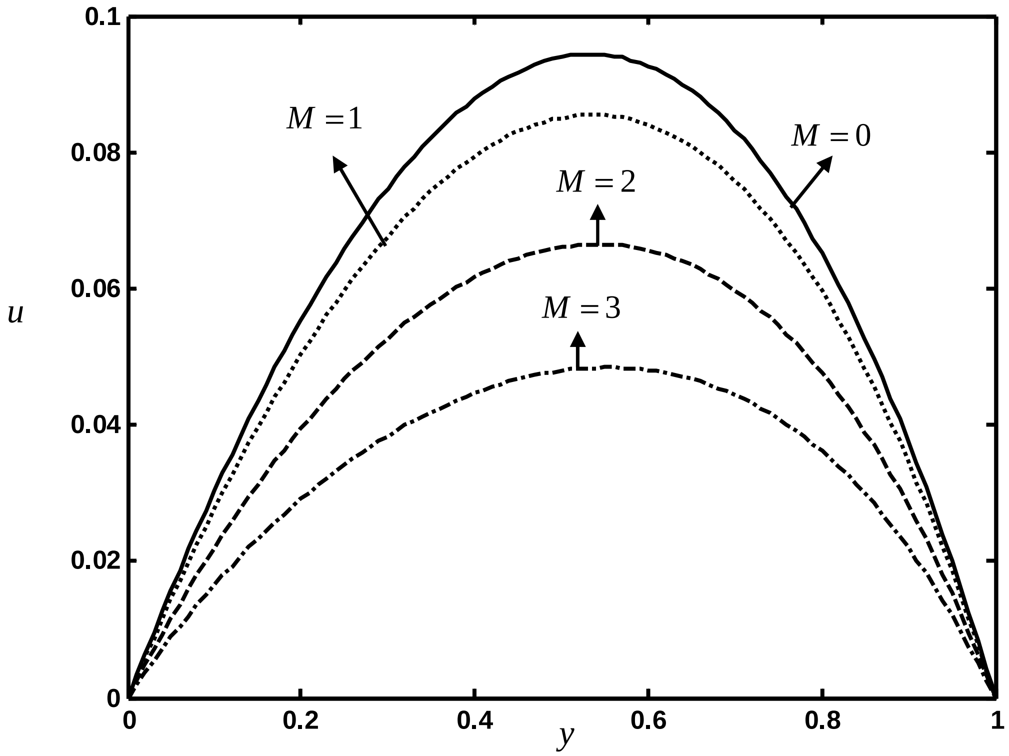

Fig. 3. Effect of Hartmann number M **on** u **for** $We=0.1, r_T=0.5$, $Gr=1$ **and** $\mathrm{Re}=1$.

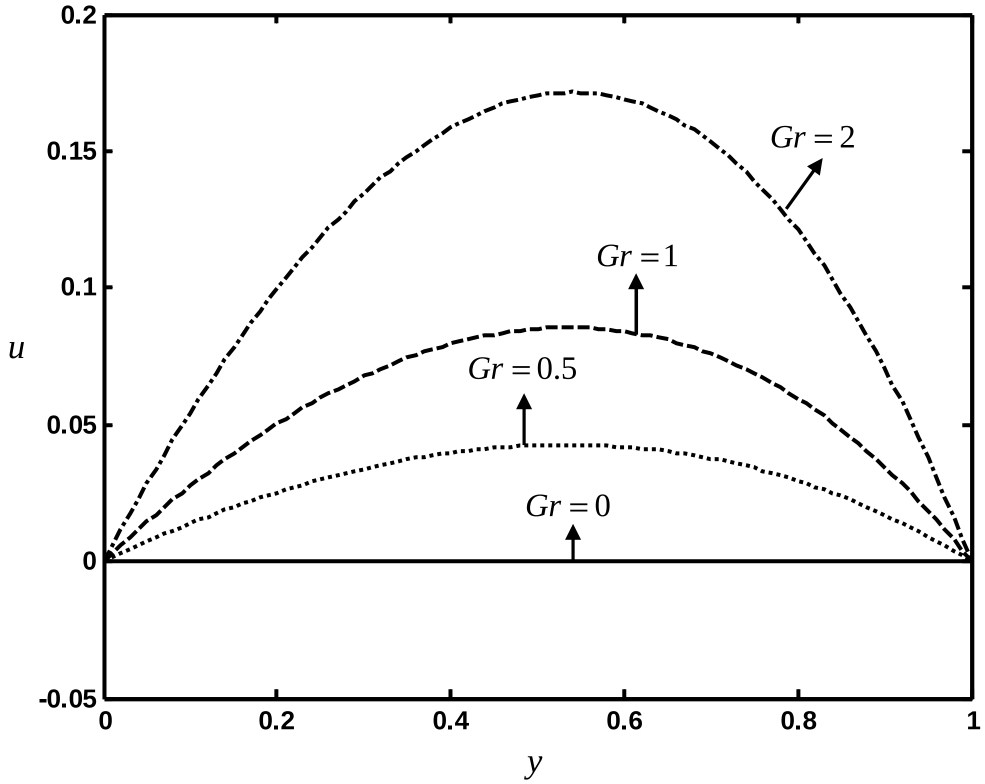

Fig. 4. Effect of Grashof number Gr **on** u **for** $M = 1, r_T = 0.5, We = 0.1$ **and** $\text{Re} = 1$.

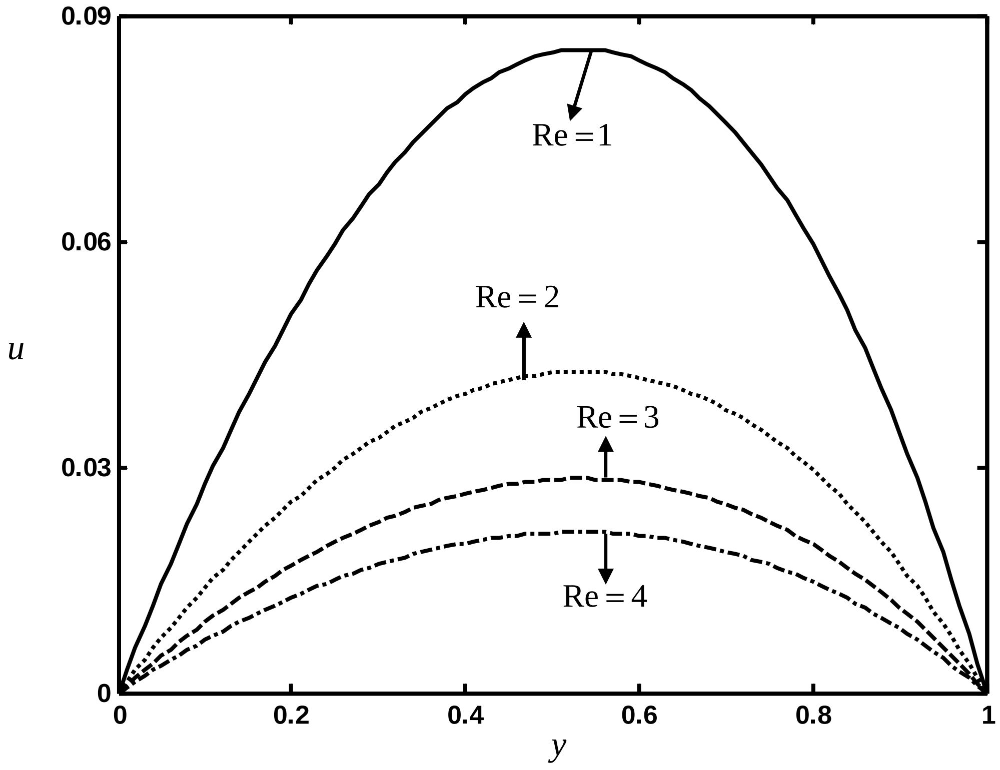

Fig. 5. Effect of Reynolds number Re **on** u **for** $M = 1, r_T = 0.5, We = 0.1$ **and** $Gr = 1$.

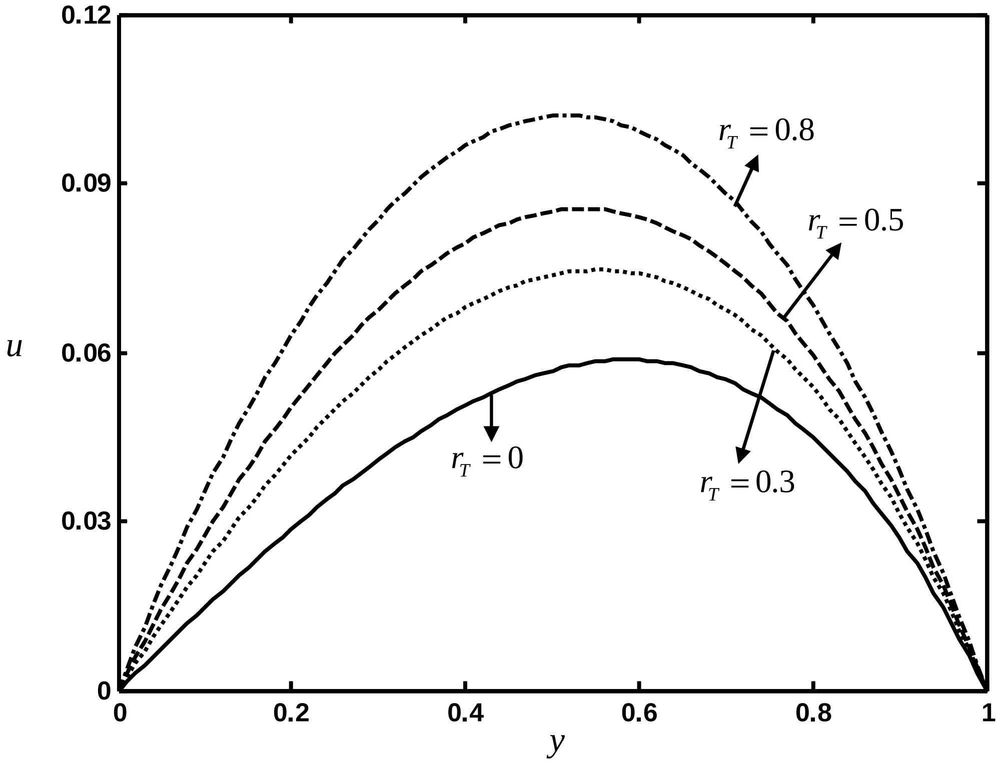

Fig. 6. Effect of wall temperature parameter r_T **on** u **for** $M = 1,\ Gr = 1, We = 0.1$ **and** $\mathrm{Re} = 1$.

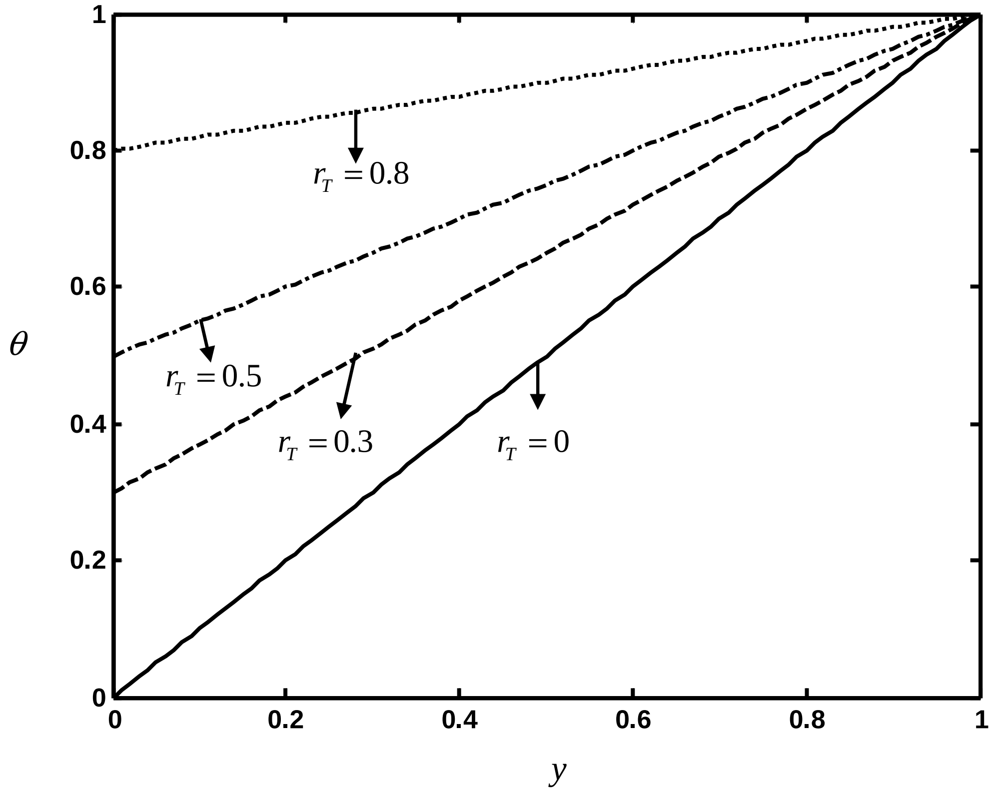

Fig. 7. Effect of wall temperature parameter r_T **on** θ .

Fig. 4 depicts the effect of Grashof number Gr on u for $M = 1, r_T = 0.5,\ We = 0.1$ and $\mathrm{Re} = 1$. It is observed that, the velocity u increases with increasing Grashof number Gr .

The effect of Reynolds number Re on u for $M = 1, r_T = 0.5, Gr = 1$ and $We = 0.1$ is shown in Fig. 5. It is noted that, the velocity u decreases with an increase in Reynolds number Re.

Fig. 6 illustrates the effect of wall temperature parameter r_T on u for $M = 1,\ We = 0.1,\ Gr = 1$ and $\mathrm{Re} = 1$. It is found that, the velocity u increases with increasing r_T .

Fig. 7 shows the effect of wall temperature parameter r_T on θ. It is observed that, the temperature θ increases with an increase in r_T .

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