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The radiation effect on the unsteady MHD convection flow through a non-uniform horizontal channel

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ABSTRACT

In this chapter, we discuss the radiation effect on the unsteady MHD convection flow through a non-uniform horizontal channel. The unsteadiness is due to the imposed oscillatory flux on the convection flow through the non-uniform channel. The perturbation analysis is carried out with the slope of the boundary as the perturbation parameter. The velocity and temperature profiles were plotted and their behavior is discussed in detail. The Stress and the average Nusselt number are also calculated and tabulated for these sets of parameters.

Key Words: MHD, Porous Medium, Prandtl Number, Grashof Number and Radiation Parameter.

INTRODUCTION

Unsteady convection flows play an important role in aerospace technology, turbo-machinery and chemical Engineering. Such flows arise due to either unsteady motion of boundary or boundary temperature. Unsteadiness may also be due to oscillatory free stream velocity or temperature. These oscillatory free convective flows are important from technological point of view. Nanda and Sharma [1, 2] have discussed the unsteady free convective flow past a semi-infinite plate with oscillatory wall temperature and shown the existence of similarity solution. Later Soundalgekar and Pop [3] have solved this problem using momentum-integral method. Kelleher and Yang [4] have studied different aspects of this problem. They obtained similar solutions of the laminar free convection boundary layer equations for the inner and the outer steady flow along a vertical heated plate whose temperature oscillates, when the mean surface temperature varies as power 'n' of distance from the leading edge. The corresponding semi-infinite horizontal plate whose temperature oscillates about a constant mean has been studied by Muhuri and Maity [5] and Mital [6]. Merkin [7] and Zeytonian [8] have also analyzed free convection effects on an infinite horizontal cylinder, when its temperature oscillates harmonically with time. Recently two problems on free convection have been solved by Pop [9, 10]. Muhuri and Maity [5] have considered the free convection flow and heat transfer along a semi-infinite horizontal plate when plate temperature oscillates about a constant mean. Verma and Singh [11] have analysed the free convection flow along a horizontal plate oscillatory in its own plane. The effects of surface temperature oscillations on the skin friction and the heat transfer from a surface to the surrounding flow is of special interest to the heat transfer engineering. The effect of plate temperature oscillations on free convection flow along the semi-infinite horizontal plate has been considered by Sharma and Mishra [12] based on Lighthill's technique and the steady state solutions were obtained using Karman-Poulhasen method.

Vajravelu and Nayfeh [13] have investigated the influence of the wall wavyness on friction and pressure drop of the generalized couette flow. Vajravelu and Sastry [14] have analysed the free convective heat transfer in a viscous, incompressible fluid confined between long vertical wavy wall and a parallel flat wall in the presence of a constant heat source. Later Vajravelu and Debnath [15] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations.

2. Formulation of the problem

We consider the unsteady motion of a viscous, incompressible electrically conducting fluid through a porous medium in a horizontal channel bounded by wavy walls in the presence of a constant heat source /sink. A uniform magnetic field of strength 'Ho' is applied normal to the walls. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy and Ohmic dissipations are neglected in comparison to the flow by conduction and convection . Also the kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system O (x ,y) with x-axis in the direction of motion and y-axis in the vertical direction and the walls are taken at y = ± Lf(δx/L), where 2L is the distance between the walls, f is a twice differentiable function and δ is a small parameter proportional to the boundary slope. A linear density temperature variation is assumed with ρ_e and T_e are the density and temperature in the equilibrium state. The flow is maintained by an oscillatory volume flux rate for which a characteristic velocity is defined as

$$q(1+ke^{i\varpi t}) = (\frac{1}{L})\int_{-Lf}^{Lf} u\,dy \qquad \textbf{(2.1)}$$

The equations governing the unsteady magneto hydrodynamic flow and heat transfer in Cartesian coordinate system O(x,y,z), in the absence of any input electric field are

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \textbf{(2.2)}$$

Equation of linear momentum

$$\rho_e(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-(\frac{\sigma\mu_e^2 H_o^2}{\rho_o})u \qquad \textbf{(2.3)}$$

$$\rho_e(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\rho g \qquad \textbf{(2.4)}$$

Equation of energy

$$\rho_e C_p(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}) = \lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+Q-\frac{\partial(q_r)}{\partial y} \qquad \textbf{(2.5)}$$

Equation of state

$$\rho-\rho_e = -\beta\rho_e(T-T_e) \qquad \textbf{(2.6)}$$

where 'ρ_e' is the density of the fluid in the equilibrium state, 'T_e' is the temperature and in the equilibrium state, (u,v) are the velocity components along O(x,y) directions, 'p' is the pressure, 'T' is the temperature in the flow region, 'ρ' is the density of the fluid, 'μ' is the constant coefficient of viscosity,

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x}-\rho_e g \qquad \textbf{(2.7)}$$

Where $p = p_e + p_D, p_D$ being the hydrodynamic pressure and in this state the temperature gradient balances the heat flux generated by source Q.

The boundary conditions for the velocity and temperature fields are

$u=0,\ v=0, T=T_1(\eta)$ on y = -L f(δx/L)

$u=0,\ v=0, T=T_2(\eta)$ on y = L f(δx/L) **(2.8)**

Invoking Rosseland approximation (Brewester(1a)) for the radiative flux we get

$$q_r = \frac{4\sigma}{3\beta_R}\frac{\partial(T'^4)}{\partial y} \quad \textbf{(2.9)}$$

expanding T'^4 in Taylor series about T_e and neglecting higher order terms (19a),

$$T'^4 \approx 4TT_e^3 - 3T_e^4 \quad \textbf{(2.10)}$$

In view of the continuity equation (**2.3**) we define the stream function ψ as

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = -\frac{\partial\psi}{\partial x} \quad \textbf{(2.11)}$$

Eliminating pressure p from equations (**2.3**) & (**2.4**) and using (**2.11**) the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi + \beta g(T-T_0)_x - (\frac{\sigma\mu_e^2 H_o^2}{\rho_e})\frac{\partial^2\psi}{\partial y^2} \quad \textbf{(2.12)}$$

$$\rho_e C_p(\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}) = k_1\nabla^2\theta + Q + \frac{16T_e^3\sigma^*}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \quad \textbf{(2.13)}$$

Introducing the non-dimensional variables in (**2 .12**) & (**2.13**) as

$$x' = x/L,\ y' = y/L, t' = t\varpi, \Psi' = \Psi/qL, \theta = \frac{T-T_e}{T_2-T_e} \quad \textbf{(2.14)}$$

The corresponding boundary conditions

$$\psi(+1) - \psi(-1) = (1 + ke^{i\varpi t})$$

$$\frac{\partial\psi}{\partial x} = 0,\ \frac{\partial\psi}{\partial y} = 0 \quad , \theta = \frac{T_1 - T_e}{T_2 - T_e} = h, say \qquad on\ y = -f(\delta x)$$

$$\frac{\partial\psi}{\partial x} = 0,\ \frac{\partial\psi}{\partial y} = 0 \quad , \theta = 1 \qquad on\ y = f(\delta x)$$

$$\frac{\partial\theta}{\partial y} = 0, \quad at \quad y = 0 \quad \textbf{(2.15)}$$

Where

$R = \frac{qL}{\nu}$ (the Reynolds number)

$G = \frac{\beta g(T_2 - T_e)L^3}{\nu^2}$ (the Grashof number)

$P = \frac{\mu C_p}{k_1}$ (the Prandtl number)

$\alpha = \frac{QL^2}{k}$ (the Heat source parameter)

$N = \frac{4\sigma^* T_e^3}{\beta_R k_1}$ (the radiation parameter)

3. Solution of the problem

Introduce the transformation such that

$$\bar{x} = \delta x \quad , \quad \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \bar{x}}$$

Then

$$\frac{\partial}{\partial x} \sim O(\delta) \;\rightarrow \frac{\partial}{\partial \bar{x}} \sim O(1)$$

For small values of δ<<1 the flow develops slowly with axial gradient of order δ and hence we take $\frac{\partial}{\partial \bar{x}} \sim O(1)$.

We adopt the perturbation scheme and write

$$\psi(x,y) = (\psi_0 + ke^{it}\bar{\psi}_0) + \delta(\psi_1 + ke^{it}\bar{\psi}_1) + --------$$

.

$$\theta(x,y) = (\theta_0 + ke^{it}\bar{\theta}_0) + \delta(\theta_1 + ke^{it}\bar{\theta}_1) + ---------$$

The corresponding boundary conditions are

$$\theta_1(+1) = 0, \qquad \theta_1(-1) = 0$$

$$\psi_1(+1) - \psi_1(-1) = 0, \psi_{1,\eta}(\pm 1) = 0, \psi_{1,x}(\pm 1) = 0$$

$$\bar{\theta}_1(\pm 1) = 0,$$

$$\bar{\psi}_1(+1) - \bar{\psi}_1(-1) = 0, \bar{\psi}_{1,\eta}(\pm 1) = 0, \qquad \bar{\psi}_{1,x}(\pm 1) = 0$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)} \left(\frac{\partial \theta}{\partial y}\right)_{\eta=\pm 1}$$

where

$$\theta_m = 0.5 \int_{-1}^{1} \theta \, dy$$

and the corresponding expressions are

DISCUSSION OF THE NUMERICAL RESULTS

The primary aim of our analysis is to investigate the radiation effect on the behavior of the temperature induced buoyancy force taking in to account the effect of surface geometry and wall temperature ratio. The flow is analysed for different sets of the parameters G,R,M,β,α,γ and N_1 governing the flow. It should be noted that the flow is basically asymmetric due to distinct surface temperatures. For computation purpose we assume the boundaries to be $y = \pm f(\bar{x}) = \pm(1 + \beta e^{-x^{-2}})$ and β>0 corresponds to dilated channel and β<0 corresponds to constricted channel. The transformation $\eta = \frac{y}{f(\bar{x})}$ reduce the boundaries to η=±1. We confine our attention to dilated channel. The non-uniformity in the boundary gives rise to the secondary transverse flow and hence the general pattern of the flow can be judged by the behavior of the Ressultant of primary and secondary velocities. The computation of the individual velocity components would enable us to investigate the effect of each body force acting on the flow and its related influence on the primary and secondary flows.

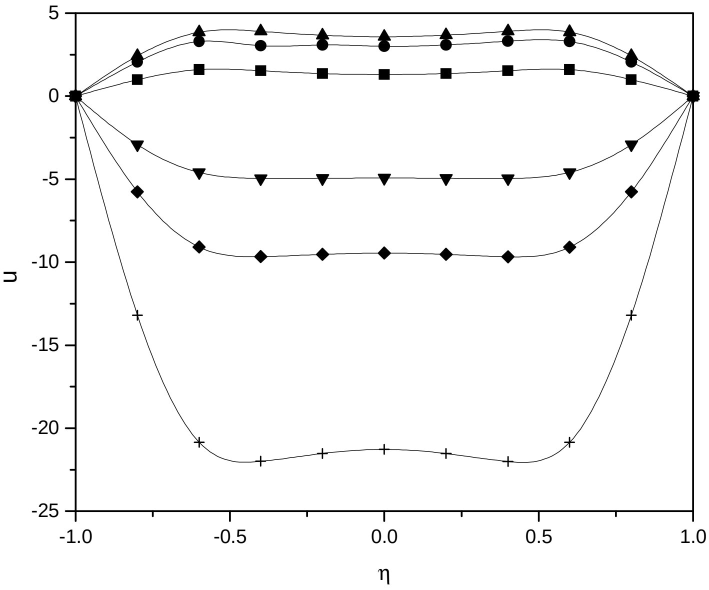

Fig [1] Variation of u with G
R=35, M=2, α=2, β=0.5, N_1=4, x=π/4, t=π/4

I	**II**	**III**	**IV**	**V**	**VI**	
G	**$5x10^2$**	**10^3**	**$3x10^3$**	**$-5x10^2$**	**-10^3**	**$-3x10^3$**

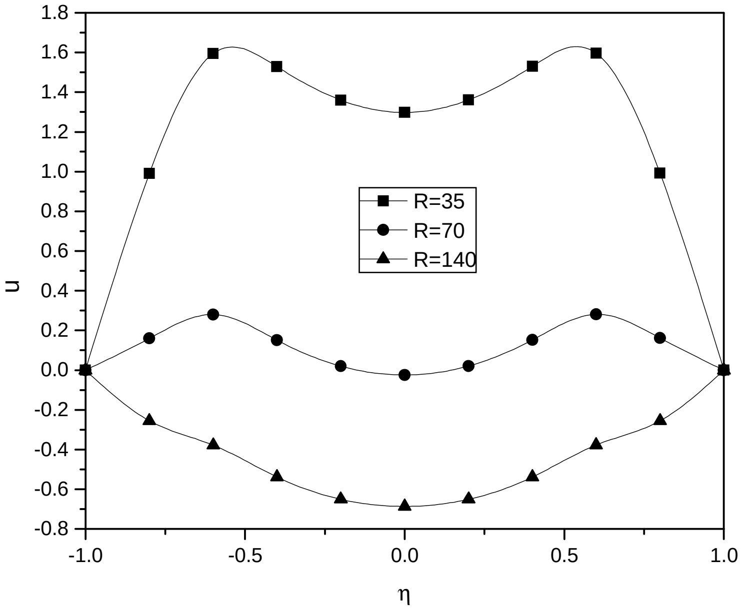

Fig [2] Variation of u with R
G=$5x10^2$, M=2, α=2, β=0.5, N_1=4, x=π/4, t=π/4

	I	**II**	**III**
R	**35**	**70**	**140**

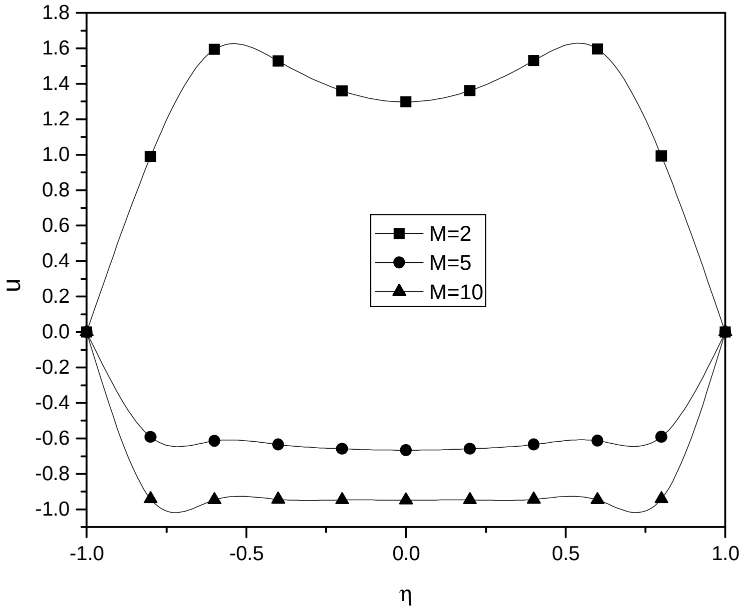

Fig [3] Variation of u with M

$G=5x10^2$, R=35, α=2, β=0.5, $N_1=4$, x=π/4, t=π/4

	I	II	III
M	**2**	**5**	**10**

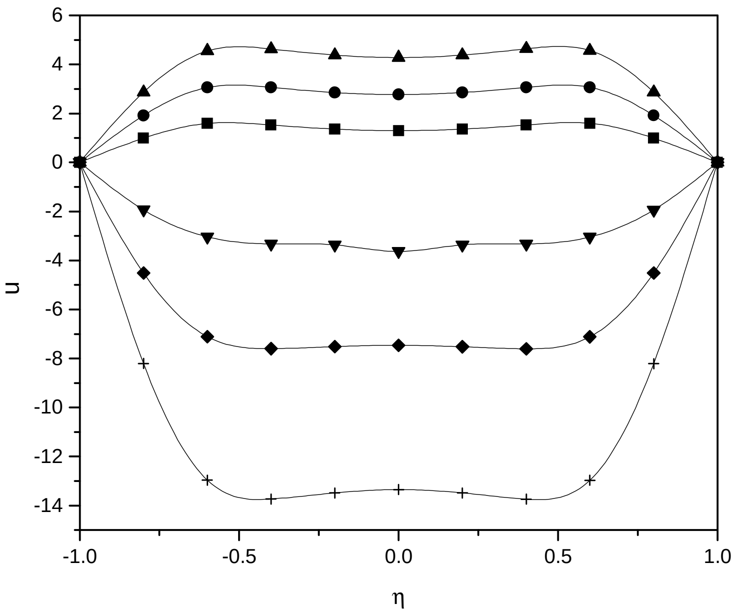

Fig [4] Variation of u with α

$G=10^2$, R=35,M=2, β=0.5, $N_1=4$, x=π/4, t=π/4

	I	II	III	IV	V	VI
α	**2**	**4**	**6**	**-2**	**-4**	**-6**

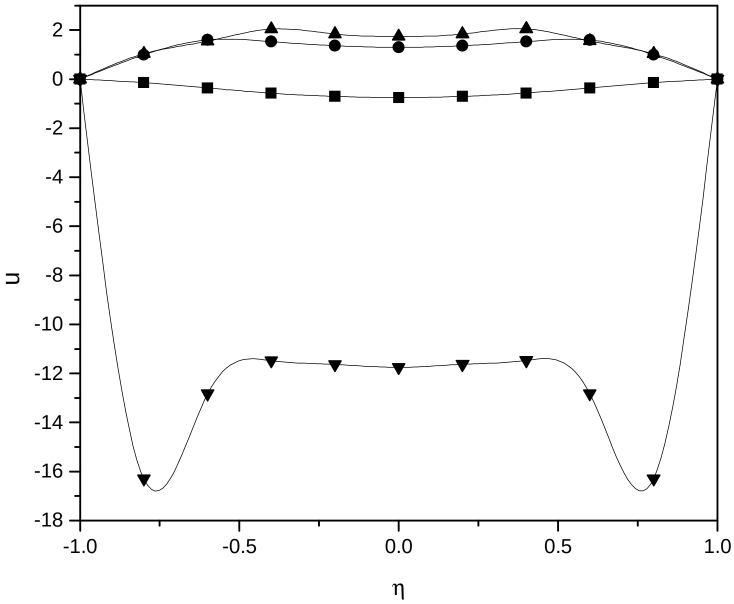

Fig [5] Variation of u with β
$G=5x10^2$, R=35, M=2,α=2, N_1=4, x=π/4, t=π/4

	I	II	III	IV
β	**0.3**	**0.5**	**0.7**	**0.9**

As in the case of resultant flow the primary velocity u is positive, for all G>0. In the case of cooling of the channel walls we find that the velocity changes from positive to negative near the lower boundary η=-1 there by exhibiting a reversal flow for $|G|=10^3$, and for higher values of |G| we notice the reversal flow in the entire flow region. This region enlarges with increase in |G|(<0) with maximum occurring at η=-0.4. For G>0 the maximum of u occurs at η=0.6 and this point of maximum velocity drifts towards the mid region for higher $G\geq10^3$ (fig. (1)).

Fig. [2], shows the variation of 'u' with Reynolds number R. It is found that for a smaller value of R=35 there is no reversal flow, but for higher R=70 the reversal occurs in the midregion and for still higher values of R, the reversal flow appears in the entire flow region. |u| reduces with R≤70 and enhances for higher R≥140.

Fig. [3], indicates that the reversal flow occurs in the entire flow region for higher values of M≥5 and this enlarges with increase in M. The variation of 'u' with the heat source/sink parameter is exhibited in fig.[4]. It is noticed that for α>0 there is no reversal flow any where in the fluid region while for α<0 we notice reversal flow in the entire flow region and this enlarges with increase in |α|(<0).

The influence of surface geometry on the flow phenomena is exhibited in fig.[5]. The reversal flow which appears in entire flow region for β=0.3, disappears for higher β≤0.6 and reappears in the fluid region, for β=0.9. Higher the dilation of the channel walls larger the magnitude of u.

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