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Wet dark fluid in spherical symmetric space-time admitting conformal motion

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ABSTRACT

In this paper, we have examined the wet dark fluid matter in the spherical symmetric space- time admitting one-parameter group of conformal motions. Also, we have discussed the properties of the solution obtained.

Keywords: Spherical symmetry, wet dark fluid and conformal motion.

INTRODUCTION

The problem of determination of exact solutions of Einstein- Maxwell equations in general relativity has attracted wide attention. Pant and Sah [1] obtained an analytic solution of relativistic field equations for a static, spherically symmetric distribution of charged perfect fluid. Tikekar Ramesh [2] discussed formal features of Einstein- Maxwell equations for spherically symmetric distribution of charged perfect fluid in equilibrium and presented exact solutions of system of equations for specified choice of matter density and fluid pressure, representing a charged perfect gas. Deo [3] studied spherically symmetric Kantowski- Sachs space- time in the context of Rosen bimetric relativity with the source of matter wet dark energy. Deo [4] also studied homogeneous anisotropic Bianchi type- I universe for the matter wet dark energy component of the universe i. e. wet dark fluid in the context of bimetric relativity. Mishra and Sahoo [5] studied Kink space- time in scale invariant theory with wet dark fluid. Aktas and Yilmaz [6] examined the magnetized quark and strange quark matter in the spherical symmetric space- time admitting conformal motion.

General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein equations. Symmetries of geometrical/ Physical relevant quantities of this theory are known as collineations and the most useful collineation is conformal killing vector defined by

$$\pounds_{\xi}\, g_{ij} = \xi_{i;j} + \xi_{j;i} = \psi\, g_{ij}, \qquad \psi = \psi(x^i),$$

where $\pounds_{\xi}$ signifies the Lie derivative along ξ^i and $\psi=\psi(x^i)$, is the conformal factor. In particular, ξ is a special conformal killing vector, if $\psi_{;ij}=0$ and $\psi_{,i} \neq 0$. Here (;) and (,) denote covariant and ordinary derivatives respectively.

The paper is outlined as follows:
In Sec.2, we have obtained Einstein field equations for static spherically symmetric distribution of wet dark fluid admitting one-parameter group of conformal motions. In Sec.3, the solutions of the Einstein field equations are obtained for wet dark fluid. At the end, the properties of the solution obtained are discussed in concluding section.

MATERIALS AND METHODS

2. Field Equations

The most general static line element with spherical symmetry is given by

$$ds^2 = -e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + e^{\upsilon}dt^2, \tag{1}$$

where λ and υ are functions of r alone and $x^{1,2,3,4} = r, \theta, \phi, t$.

Also, Einstein field equations can be expressed as

$$R_{ij} - \frac{1}{2}R\, g_{ij} + \Lambda\, g_{ij} = -8\pi\, T_{ij} \tag{2}$$

where T_{ij} is energy momentum Tensor for wet dark fluid (WDF) and Λ is the cosmological constant.

We have

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_i u_j + p_{WDF}\, g_{ij} \tag{3}$$

together with

$$g_{ij}u^i u^j = 1, \tag{4}$$

where u^i is the four- velocity vector of the fluid , p_{WDF} and ρ_{WDF} are the pressure and energy density of wet dark fluid respectively.

Here, we shall use geometrized units so that $8\pi G = c = 1$.
Then using line element (1), from (2) and (3), we get

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} - \Lambda = \rho_{WDF,} \tag{5}$$

$$e^{-\lambda}\left(\frac{\upsilon'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} + \Lambda = p_{WDF} \tag{6}$$

and

$$\frac{e^{-\lambda}}{2}\left(\upsilon'' + \frac{\upsilon'^2}{2} + \frac{\upsilon' - \lambda'}{r} - \frac{\upsilon'\lambda'}{2}\right) + \Lambda = p_{WDF} \tag{7}$$

where primes denote differentiation w. r. t. r.

Now, we shall assume that space- time admits a one- parameter group of conformal motions (Aktas and Yilmaz [6]) i.e.

$$£_{\xi}\, g_{ij} = \xi_{i;j} + \xi_{j;i} = \psi\, g_{ij}, \tag{8}$$

where $£_{\xi}$ signifies the Lie derivative along ξ^i and ψ is an arbitrary function of r. In particular, ξ is a special conformal killing vector, if $\psi_{;ij} = 0$ and $\psi_{,i} \neq 0$. Here (;) and (,) denote covariate and ordinary derivatives respectively.

Conformal killing vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations.

Using (1) and (8) by virtue of spherical symmetry, we get the following expressions:

$$\xi^1 \upsilon' = \psi, \tag{9}$$

$$\xi^4 = c_1 = constant, \tag{10}$$

$$\xi^1 = \frac{\psi r}{2}, \tag{11}$$

and

$$\lambda'\xi^1 + 2\xi^1{}_{,1} = \psi, \tag{12}$$

where a comma denotes partial derivatives.

Using equations (9) to (12), we obtain

$$e^{\nu} = c_2^{\,2} r^2, \tag{13}$$

$$e^{\lambda} = \left(\frac{c_3}{\psi}\right)^2 \tag{14}$$

and

$$\xi^i = c_1\delta_4^i + \left(\frac{\psi r}{2}\right)\delta_1^i \tag{15}$$

where c_2 and c_3 are the constants of integrations.

Substituting (13) and (14) into (5) - (7), we get

$$\frac{1}{r^2}\left(1 - \frac{\psi^2}{c_3^2}\right) - \frac{2\psi\psi'}{c_3^2 r} - \Lambda = \rho_{WDF,} \tag{16}$$

$$\frac{1}{r^2}\left(1 - \frac{3\psi^2}{c_3^2}\right) - \Lambda = -p_{WDF,} \tag{17}$$

and

$$\frac{\psi^2}{c_3^2 r^2} + \frac{2\psi\psi'}{c_3^2 r} + \Lambda = p_{WDF}. \tag{18}$$

RESULTS AND DISCUSSION

3. Solutions of Field Equations

From (17) and (18), we get

$$2\psi\psi' - \frac{2}{r}\psi^2 = -\frac{c_3^2}{r}. \tag{19}$$

Putting $\psi^2 = z$, we get

$$\frac{dz}{dr} - \frac{2}{r}z = -\frac{c_3^2}{r}. \tag{20}$$

This is the first order linear differential equation having solution

$$z = \frac{c_3^2}{2} - kr^2, \tag{21}$$

where k (> 0) is the constant of integration.

Therefore the general solution of equation (19) is given by

$$\psi^2 = \frac{c_3^2}{2} - kr^2. \tag{22}$$

Subtracting (16) and (17), we get

$$\rho_{WDF} + p_{WDF} = \frac{2\psi^2}{c_3^2 r^2} - \frac{2\psi\psi'}{c_3^2 r} = \frac{1}{r^2}. \tag{23}$$

Subtracting (16) and (18), we get

$$\rho_{WDF} - p_{WDF} = \frac{1}{r^2} - \frac{2\psi^2}{c_3^2 r^2} - \frac{4\psi\psi'}{c_3^2 r} - 2\Lambda = \frac{6k}{c_3^2} - 2\Lambda, \quad (24)$$

From (23) and (24), we obtain

$$\rho_{WDF} = \frac{1}{2r^2} + \frac{3k}{c_3^2} - \Lambda \quad (25)$$

and

$$p_{WDF} = \frac{1}{2r^2} - \frac{3k}{c_3^2} + \Lambda. \quad (26)$$

Using (13) and (14), the space-time geometry of wet dark fluid (i.e. the line element given by (1)) becomes

$$ds^2 = -\frac{c_3^2}{\psi^2} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + c_2^2 r^2 dt^2. \quad (27)$$

The equation of state for Wet Dark Fluid (WDF) is given by

$$p_{WDF} = \gamma(\rho_{WDF} - \rho^*), \quad (28)$$

where the parameters γ and ρ^* are taken to be positive and $0 \leq \gamma \leq 1$, and it is good approximation for many fluids including water, where the internal attraction of the molecules make negative pressure possibly.

If we set $\Lambda = \frac{3k}{c_3^2}$, then for $\rho^* = 2\Lambda - \frac{6k}{c_3^2}$ and γ =1, the equation of state for WDF (28) is satisfied.

Further

$$p_{WDF} = \rho_{WDF}\,. \quad (29)$$

WDF has two components:

One behaves as a cosmological constant and other as standard fluid with equation of state $p = \gamma\rho$ (Mishra and Sahoo [5]).

Further, from (23)

$$\rho_{WDF} + p_{WDF} = \frac{1}{r^2} \geq 0. \quad (30)$$

This implies that WDF will not violate the strong energy condition.

CONCLUSION

From (25) and (26), it is clear that ρ_{WDF} and p_{WDF} are not regular at centre of sphere ($r = 0$) and we do not have finite radius of the sphere for which $p_{WDF} = 0$.

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